

# Data-driven Uncertainty and Sensitivity Analysis for Ship Motion Modeling in Offshore Operations

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## Abstract

To build a compact data-driven ship motion model for offshore operations that require high control safety, it is necessary to select the most influential parameters and to analyze the uncertainty of the input parameters. This paper proposes a framework of uncertainty and sensitivity analysis for ship motion data. The framework consists of four components: data cleaning, surrogate model, sensitivity and uncertainty analysis, and results visualization. Data cleaning focuses on the removal of noise, and necessary transformation for the easy analysis. An artificial neural network (ANN) based surrogate model is constructed on the basis of cleaned ship motion data. The sensitivity and uncertainty analysis would be performed on the sample or weights which the ANN based surrogate model generated. The result of the sensitivity and uncertainty analysis can be beneficial to the optimization of data-driven ship motion models. Three distinctive sensitivity analysis (SA) methods (Garson/Morris/Sobol), and PDF-based and CDF-based uncertainty methods are investigated in two types of ship motion datasets with and without environmental factors. The experimental results also demonstrate the proposed framework can be applied to estimate the sensitivity and uncertainty in different datasets.

*Keywords:* data-driven, uncertainty analysis, sensitivity analysis, ship motion, offshore operations

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## 1. Introduction

With the rapid development of the offshore industry, the requirements for ship operations are increasing. Offshore operations are complex and hazardous due to the significant uncertainties and various operating conditions. It is necessary to consider the accuracy of ship positioning and heading, limited working space, and collision avoidance between ships and floating structures [1]. Operational safety is the major issue and is easily challenged by harsh marine environments, complex geological conditions, and human and equipment factors. Operations around the oil drilling platform require very careful manoeuvring of ships near the target and collision avoidance in designated areas, and maneuvering in the limited space does not demand a fast approach, but requires a high degree accuracy of positioning and heading. Thus, establishing a suitable ship motion model for offshore operations is necessary.

To get a suitable model of ship motion in offshore operations, the conventional method is to employ the mathematical modeling [2]. However, it is not an easy task to model the ship motion during the offshore operations, because the model should be nonlinear, dynamic over time, and coupling with environmental factors, like currents, waves, and wind which are random and unpredictable. Thus, the conventional model-based solutions, which require an in-depth knowledge of the offshore operations, are impractical for complicated offshore operations. With the wide deployment of sensors on ships, it is possible to collect ship motion data and establish data-based models.

The greatest challenge of data-driven modeling for offshore operations is that environmental factors such as the current, wave, and wind tend to influence them. The high uncertainty of environmental factors increases the difficulty of modeling. Thus, studying the influence of environmental factors on the data-based models stands to vastly improve safety as well as profit margins. However, to the best knowledge of the authors, related researches on the impact of environmental factors on data-driven modeling is still rare. On the other hand, there are also some other challenges for the data-driven modeling

for offshore operations: 1) the sensor data used to be modelled are usually too large and high dimensional; 2) they usually contain measurement-induced noise, redundant information, and human factors, which makes it difficult to use them for accurate analysis; 3) it is not intuitive to interpret the data from multiple sensors. As an aspect of uncertainty quantification, the sensitivity analysis is defined as the investigation of “how the uncertainty of the model output is apportioned to different sources of uncertainty in the model input factors” [3]. SA is useful for obtaining reliable results and valuable information, and increases the credibility of model results [4, 5].

SA has been widely used for maritime applications with different purposes, mainly including assessing the uncertainty, calibrating the model, and making robust decisions. A derivative-based SA is used to simplify the three-layer structure of Nonlinear Autoregressive Exogenous (NARX) neural network for ship motion prediction [6]. To construct a compact data-driven ship motion prediction model, Zhang et al. utilized the sum of square derivatives (SSD) to choose the inputs for the nonlinear autoregressive model with exogenous inputs (NARMAX) [7]. Panagiotis proposed an SA to investigate various performance parameters affecting ship propulsion and maneuverability [8]. The influence of hydrodynamic coefficients on the manoeuvrability of submersibles is examined using SA on the basis of sea trials [9]. To study the influence of hydrodynamic coefficients for a four-DOF mathematical ship model, an SA is performed on the basis of simulated data [10]. To determine the effect of the hydrodynamic derivatives on KVLCC2 maneuverability, SA is conducted on the Abkowitz-type mathematical model [11]. Uncertainty analysis of the circular motion test data was presented in [12] to investigate the hydrodynamic characteristics of ship maneuvering. Vadim et al. provided a statistical uncertainty of ship motion data [13].

Although SA methods have been widely used in the analysis of mathematical models in the field of marine engineering, no scheme has emerged that explains the uncertainty and sensitivity analysis of data-driven models in this field. Our ongoing project aims to develop intelligent systems to support decision-making

for various maritime operations. A new integrated platform, including data analysis tools and data-driven modeling technique, is designed to serve the maritime industry by improving operational efficiency and safety. In this paper, we focus on the design of framework of uncertainty and sensitivity analysis for ship motion data in offshore operations. To overcome the challenges of data-driven modeling, we first clean the raw data. Because SA methods cannot be applied to the sensor data directly, a surrogate model is constructed. Uncertainty and sensitivity analysis is then performed on the basis of surrogate model. To investigate the performance of the proposed framework, three SA methods are compared in terms of importance ranking of the input parameters on two ship motion datasets with and without environmental factors. The main contributions of this paper are: first, it proposes a new uncertainty and sensitivity analysis framework, making the SA methods be applicable to the analysis of ship motion datasets; second, it investigates the performance of different SA methods on different ship motion datasets in offshore operations.

The rest of the paper is organized as follows. Section 2 introduces the proposed framework for sensitivity and uncertainty analysis. The methods to quantify the sensitivity and uncertainty are described in Section 3. The proposed framework is examined in Section 4 on the two ship motion datasets with and without environmental factors, and the uncertainty analysis is provided after the sensitivity analysis. Finally discussion and conclusions are given.

## **2. Analysis of uncertainty and sensitivity on ship motion data**

This section introduces (1) the proposed system framework for sensitivity and uncertainty analysis; (2) the source of sensor data, data pre-processing algorithm, and the case ship; (3) data -driven surrogate modeling methods; and (4) the concept of sensitivity and uncertainty and the working flow of the proposed framework.

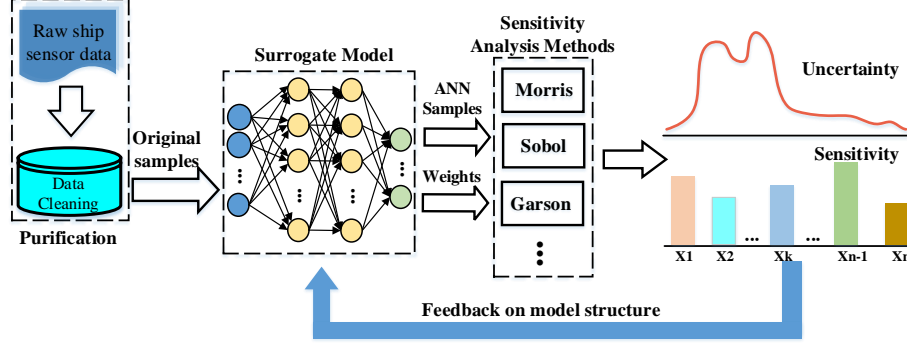


Figure 1: Framework of sensitivity and uncertainty analysis.

### 2.1. System framework

A framework that integrates surrogate modeling technique, uncertainty and sensitivity analysis methods to perform the sensitivity and uncertainty analysis is proposed. The whole framework consists of four components: data cleaning, surrogate modeling, sensitivity methods, and results analysis, which is depicted in Fig. 1. First, the raw sensor data should be cleaned to minimize the effect of noisy and redundant information. Second, surrogate-based (also called meta-based) methods offer an analytic approach to construct a mathematical model or prediction model from those sensor data. Third, taking advantage of the surrogate model, various uncertainty and sensitivity analysis methods can be performed for comparison study. Our framework is flexible such that we can employ surrogate models and SA methods to calculate the sensitivity and uncertainty results. Finally, the sensitivity and uncertainty results can be employed to optimize the surrogate structure, as the feedback arrow shown in Fig. 1.

### 2.2. Data source and data pre-processing

The Offshore Simulator Centre AS (OSC) is an advanced training platform for offshore operation personnel. The OSC is developed by the Norwegian University of Science and Technology (NTNU) and other research partners from

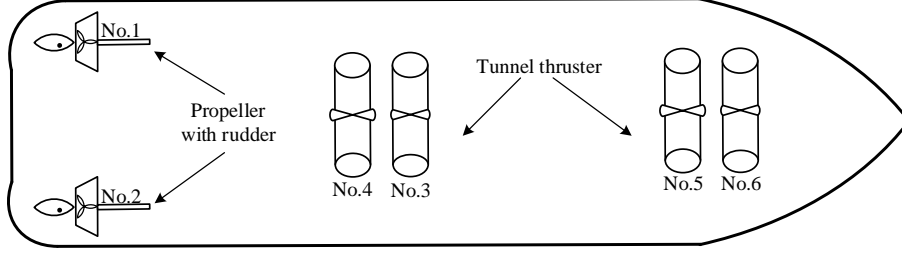


Figure 2: Case Ship.

the industry world. OSC has very powerful physical engine that can produce wind and waves that are almost the same as the real environment [14]. Fig. 2 shows the case ship equipped with two propellers with rudders and two tunnel thrusters at the stern, and two tunnel thrusters at the bow.

The first step for analysis of sensitivity and uncertainty is to obtain a starting subset and to focus on specific parameters. Selecting those parameters is highly subjective, which is based on the expert knowledge of the authors and a broad literature research. In this context, two data modules are monitored and stored: the ship status module, and the environment module, as shown in Table 1.

Considering that the original raw data may contain noisy, discontinuous and redundant information, it is necessary to clean up the data to minimize its impact on further analysis and modeling [15]. Noise reduction is always the first step in data cleaning, which statistical estimation or median filtering methods could be utilized. Due to the physical definition, there would be some jumping phenomenons in sensor data. For example, the definition of roll angle, yaw angle, and pitch angle is within  $[0^\circ, 360^\circ]$ . When the angle changes near the border, the jumping phenomenon would occur inevitably. To get rid of this type of discontinuity, the algorithm is employed which is defined in our previous paper [15]. To determine the effect of rudders, it is necessary to make some transformations to obtain correct estimation. In this paper, the effect of rudder is transformed to the lift force, and the calculation of the force is adopt from

Table 1: Recorded ship data specification

| Module      | Parameter        | Unit    | Description                    |
|-------------|------------------|---------|--------------------------------|
| Ship Status | Surge velocity   | [m/s]   | Velocity in surge direction    |
|             | Sway velocity    | [m/s]   | Velocity in sway direction     |
|             | Yaw velocity     | [deg/s] | Velocity in yaw direction      |
|             | Roll velocity    | [deg/s] | Velocity in roll direction     |
|             | Pitch velocity   | [deg/s] | Velocity in pitch direction    |
|             | Heading          | [deg]   | Rotation around the yaw axis   |
|             | Roll             | [deg]   | Rotation around the roll axis  |
|             | Pitch            | [deg]   | Rotation around the pitch axis |
|             | Thruster percent | [%]     | Output percent of thruster     |
|             | Thruster speed   | [RPM]   | Speed of thruster              |
|             | Thruster force   | [N]     | Magnitude of thruster force    |
|             | Thruster power   | [W]     | Consumed power of thruster     |
|             | Thruster rudder  | [deg]   | Rudder angle                   |
| Environment | Wave height      | [m]     | Height of wave                 |
|             | Wave direction   | [deg]   | Direction of wave              |

[16].

In addition, in order to accelerate the convergence speed of training and improve the accuracy of data-driven surrogate model, it is necessary to normalize all parameters. The equation for normalizing all the parameters is shown in Eq. (1).

$$\hat{\mathbf{X}} = \frac{\mathbf{X} - \mu(\mathbf{X})}{\sqrt{\delta(\mathbf{X}) + \epsilon}} \quad (1)$$

where  $\mu(\mathbf{X})$  denotes the mean of  $\mathbf{X}$ .  $\delta(\mathbf{X})$  represents the variance of  $\mathbf{X}$ .  $\epsilon$  is a very small value that makes the calculation possible when  $\mathbf{X}$  is a constant.

To synchronize these data with different sampling frequency, this paper also considers the re-sampling the data.

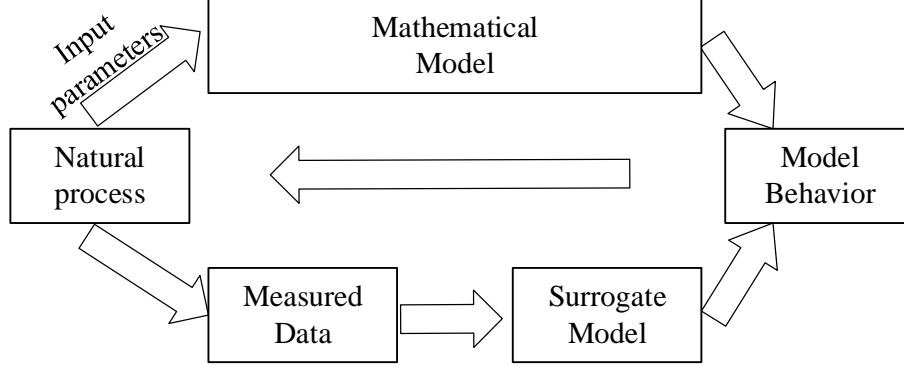


Figure 3: Concept of the surrogate model.

### 2.3. Data-driven surrogate modeling method

The process in nature usually can be simulated using some mathematical models. The input of the model  $f$  describes the material properties, external loads, boundary conditions, initial conditions, etc [17]. So that the model can describe complex natural behavior, the input dimension is usually very large and may be hundreds or thousands. It is feasible to construct a surrogate model that can reduce the computational burden of the original model while not sacrificing the accuracy. The surrogate model can be considered as “model of model”. The concept of a surrogate model is using a cheaper model to replace the original model.

Fig. 3 shows the concept of the surrogate model which can be built from the observed data of natural processes. From the given input parameters of the natural process, the mathematical model could be constructed, which then yield the relevant data. If the mathematical model is hard to build, but only the measured data is available, a surrogate model could be constructed to understand the underlying characteristics of the natural process. This is also one of the objectives of machine learning [18]. Observed data, whether from mathematical models or surrogate models, can be applied to understand natural processes.

Assuming there are  $N$  collected points, which are formulated as  $\mathcal{F} = \{\mathbf{X}, \mathbf{y}\}$ .



$\mathbf{X}$  is an  $N \times D$  matrix, which can be represented by  $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_D)$ .  $\mathbf{y}$  is the corresponding response. The surrogate model  $\hat{y} = \hat{f}(\mathbf{X})$  can be built to approximate the underlying relationship of collected data.

The widely used data-driven models, such as Kriging [19], Gaussian Process [20], the Radial Basis Function [21], support vector machine (SVM) [22], ANN model [23], and the deep learning model [17] can be effectively used for the construction of a surrogate model from measured data in our proposed framework.

In this paper, the ANN model is chosen as the surrogate model, as shown in Fig. 1. To get the best structure of ANN, many optimization approaches, such as genetic algorithm (GA) and particle swarm optimization (PSO), can be used. Furthermore, ANN also can be employed to compute the sensitivity of input factors on the basis of the “weight” of the neurons in different layers. In Fig. 1, the “Weights” is used for the ANN-based sensitivity analysis, e.g. Garson. The “ANN Samples” would be employed by performing global sensitivity analysis, such as Morris and Sobol.

#### *2.4. Sensitivity and uncertainty analysis*

The main purpose of the SA is to quantify the importance of all uncertain input parameters to a considered output. In general, SA methods can be roughly divided into local methods (LSA), which calculate quickly, but only explore some sample points selected by users in the input space, and the global method (GSA) of checking the influence of uncertain parameters on the whole parameter range [24]. The LSA and GSA have their own advantages and disadvantages and application areas. The LSA, which is computationally efficient, is more applicable to the linear models, while the GSA, which is time consuming, can be applied to the both linear and nonlinear models. Both LSA and GSA can be used in this proposed framework according to different applications. The results of SA are helpful for the removal of the less relevant inputs and optimizing the model structure [25].

Uncertainty analysis is an important and necessary step for the data-driven models. Usually, there are two types of uncertainties: aleatoric uncertainty

and epistemic uncertainty [26]. Aleatoric uncertainty is often caused by the imprecise measurement in the data, and epistemic uncertainty is caused by the parameter of the model. First, this paper focus on the uncertainty caused by each input parameter. Second, this paper also aims at the uncertainty analysis of the set of input parameters selected by the SA methods.

The process of the framework is as follows: 1) cleaning the raw ship sensor data using the algorithm described in Section 2.2; 2) training the ANN using the cleaned sensor data; 3) generating the ANN samples and obtaining the ANN weights; 4) calculating the sensitivity of different methods, which is introduced in Section 3. Finally, the sensitivity indices can help to improve the performance of the surrogate model.

### 3. Quantification of sensitivity and uncertainty

The paper mainly introduces three types of SA methods with different computational time: Garson, Morris, and Sobol. The Garson method belongs to the LSA, which is highly efficient. It is a widely used method of SA in data-based applications that is computed through the connection of different layers. The Morris method belongs to the GSA, which means it has a medium computational burden. It provides a qualitative ranking of the parameters that are based on their effect on the output of the model [27]. The Morris method provides a compromise scheme and represents a suitable GSA method for those computationally intensive models with large input parameters. The Sobol method belongs to the GSA, which is computationally expensive. The variance-based sensitivity methods are also very popular for data-driven applications in the literatures [25, 28].

#### 3.1. Garson

The Garson method is a special type of LSA that can be very useful to analyze the structure of ANN, which employs the connected weights of ANN to calculate the sensitivity of each input parameter [29]. It can provide a quick

and informative understanding of how the ANN output can be attributed to its input parameters. Concretely, the sensitivity of the  $i$ -th uncertain input parameter to the  $k$ -th output can be defined as follows:

$$S_{ik} = \frac{\sum_{j=1}^L (|\omega_{ij} v_{jk}| / \sum_{r=1}^N |\omega_{rj}|)}{\sum_{r=1}^N \sum_{j=1}^L (|\omega_{ij} v_{jk}| / \sum_{r=1}^N |\omega_{rj}|)} \quad (2)$$

where  $S_{ik}$  represents the sensitivity of the  $i$ -th uncertain input to the  $k$ -th output.  $N$  and  $L$  are the number of the neurons in the input and hidden layer.  $\omega_{ij}$  is the weight of the  $i$ -th neuron in input layer and the  $j$ -th neuron in the hidden layer;  $v_{jk}$  is the weight of the  $j$ -th neuron in the hidden layer and the  $k$ -th neuron in the output layer.

### 3.2. Morris

Morris, also known as Elementary Effect (EE), is a commonly used GSA and is usually used to identify the important uncertain input parameters in high dimensional models, rather than to quantify sensitivity exactly [5, 30].

For a given model  $f(\mathbf{X})$  with  $n$  independent input parameters  $(x_1, x_2, \dots, x_n)$ , the Morris sensitivity index of  $i$ -th uncertain input parameter is defined as:

$$EE_i = \frac{f(x_1, x_2, \dots, x_i + \Delta, x_{i+1}, \dots, x_n) - f(\mathbf{X})}{\Delta} \quad (3)$$

From the definition of Equation (3), the One-At-a-Time (OAT) design is employed. To implement the global characteristic of Morris, the OAT design should be repeated  $N$  times. To measure the sensitivity, the mean ( $\mu$ ) and standard deviation ( $\sigma$ ) are used, which can be computed by Equation (4):

$$\begin{aligned} \mu_i &= \frac{1}{N} \sum_{j=1}^N EE_i(j) \\ \sigma_i &= \sqrt{\frac{1}{N-1} \sum_{j=1}^N [EE_i(j) - \frac{1}{N} \sum_{j=1}^N EE_i(j)]^2} \end{aligned} \quad (4)$$

in which  $EE_i(j)$  is the elementary effect for input  $i$  using the  $j$ -th base sample point. The Morris method uses the  $\mu$  to estimate the significant effect on the output, and used the  $\sigma$  to estimate the non-linearity and interactions between inputs. If  $\mu_i$  is close to zero, it indicates the parameter  $i$  is non-influential. If  $\sigma_i$  is large this indicates the parameter  $i$  has a nonlinear effect on output, or that interactions with other parameters exist.

### 3.3. Sobol

The Sobol method estimates the importance of uncertain input parameter on the basis of variance decomposition, which has been widely used in many disciplines [31]. Assuming the model form is  $f(\mathbf{X}) = f(x_1, \dots, x_M)$ , where  $\mathbf{X} = (x_1, \dots, x_M)$  represents the model input which contains  $M$  independent parameters. Based on the theory of Sobol [32], the model output can be decomposed by different effects, which is shown as follows:

$$f(\mathbf{X}) = f_0 + \sum_{i=1}^M f_i(x_i) + \sum_{1 \leq i \leq j \leq M} f_{ij}(x_i, x_j) + \dots + f_{1,2,\dots,M}(x_1, x_2, \dots, x_M). \quad (5)$$

Some literatures [33, 34] have argued that only the lower order terms are important. So, this paper considers only the two higher orders. Eq. (5) can be re-written as follows:

$$f(\mathbf{X}) = f_0 + \sum_{i=1}^M f_i(x_i) + \sum_{1 \leq i \leq j \leq M} f_{ij}(x_i, x_j). \quad (6)$$

Assume the  $f(\mathbf{X})$  is square integrable. Squaring the Eq.(6) and integrating over the input space, the following equation can be obtained:

$$\int f^2(\mathbf{X}) d\mathbf{X} - f_0^2 = \sum_{i=1}^M \int f_i^2(x_i) + \sum_{1 \leq i \leq j \leq M} \int f_{ij}^2(x_i, x_j). \quad (7)$$

The left part in Eq.(7) is called the total variance:

$$V = \int f^2(\mathbf{X}) d\mathbf{X} - f_0^2 \quad (8)$$

The right part in Eq.(7) is called the partial variance:

$$\begin{aligned} V_i &= \sum_{i=1}^M \int f_i^2(x_i) \\ V_{ij} &= \sum_{1 \leq i \leq j \leq M} \int f_{ij}^2(x_i, x_j) \end{aligned} \quad (9)$$

Generally, the global sensitivity index would be described by the ratio of partial variance and total variance [35]. The first-order (main effect) and total-order sensitivity index for the  $i$ -th variable  $x_i$  can be defined by:

$$\begin{aligned} S_i &= \frac{V_i}{V} \\ S_{Ti} &= 1 - \frac{V_{\sim i}}{V} \end{aligned} \quad (10)$$

The first-order and total-order sensitivity index are widely used in Sobol. If the value of total(first)-order sensitivity index is close to zero, the parameter can be considered to be non-important.

### 3.4. Uncertainty analysis method

The common way of quantifying uncertainty is to estimate the probability distribution function (PDF) [17] and Cumulative Distribution Function (CDF) [36]. Although current researches either focus on PDF or CDF, the proposed framework takes advantages of both methods so that it can quantify the uncertainty of input parameters in two different perspectives.

Assuming the group of input parameter  $X_i$  of the data-driven model is not known exactly. The unconditional cumulative distribution function and probability distribution function of the model output are expressed as  $F_y(y)$  and  $f_y(y)$ . When the group of input parameter  $X_i$  is removed or is kept fixed, the cumulative distribution function and probability distribution function of the model output then can be represented as  $F_{y|x_i}(y)$  and  $f_{y|x_i}(y)$ . The distance  $dF$  and  $df$  between  $F_y(y)$ ,  $f_y(y)$  and  $F_{y|x_i}(y)$ ,  $f_{y|x_i}(y)$  accounts for the uncertainty of a set of parameters  $X_i$ , which can be described as follows:

$$dF_{X_i} = |F_y(y) - F_{y|x_i}(y)| \quad (11)$$

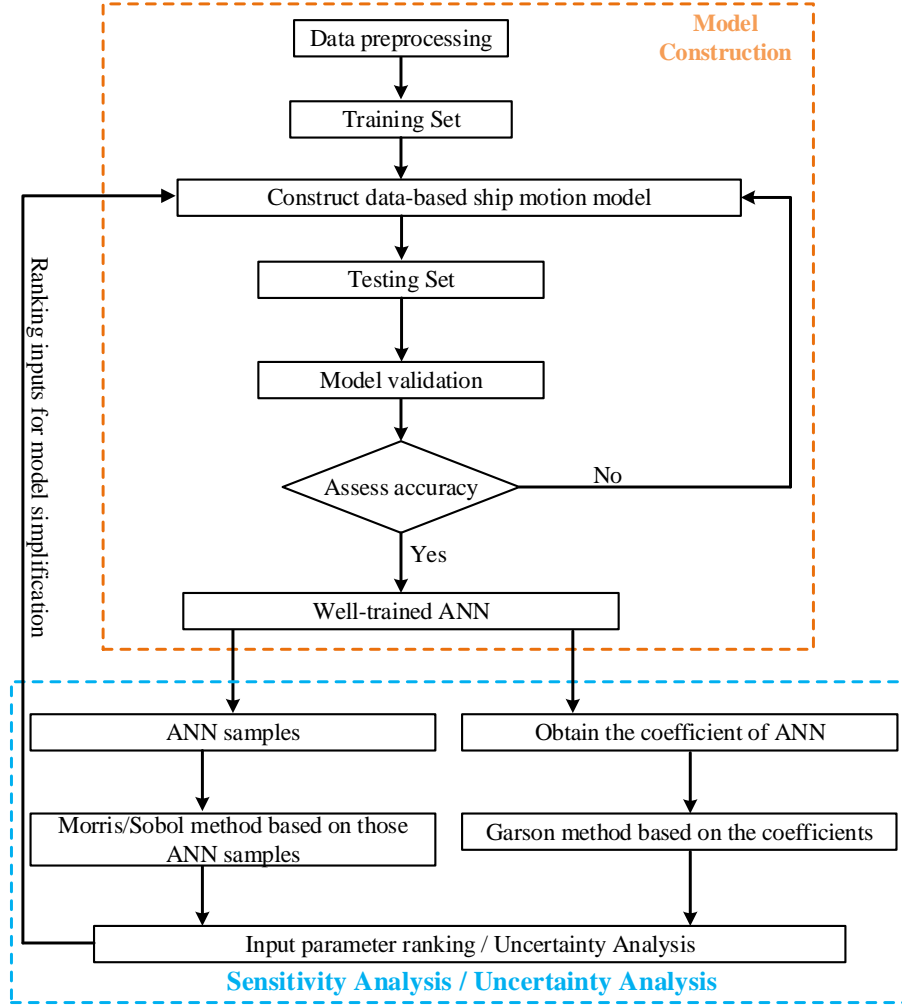


Figure 4: Workflow of the Algorithm.

$$df_{X_i} = |f_y(y) - f_{y|X_i}(y)| \quad (12)$$

### 3.5. Detail algorithm description

The algorithm used to investigate this research is illustrated in Fig. 4. First, the ship sensor data would be cleaned, using the methods as described in Sec-

tion 2.2. Second, the ANN would be built on the basis of the training set and the model would be validated by three standard evaluation measurements — mean absolute percentage error (MAPE), variance of absolute percentage error (VAPE) and root mean square error (RMSE). If the testing results are not as expected, the ANN model will need to retrain to achieve the specified accuracy. Third, the Garson sensitivity index would be calculated on the basis of the coefficient of different layers of ANN. The Morris and Sobol methods would be computed by the ANN samples. Finally, in describing the model output uncertainty, the importance of the input parameters of each SA method is quantified by sorting them in descending order.

#### 4. Experiment

In this section, we present the sensitivity and uncertainty analysis on two type of datasets of random maneuvering, with and without environment factors. For simplicity, the two datasets only records simple maneuvers in offshore operations, in which only the two main thrusters as depicted in Fig. 2, are used to control ship motion. In the dataset without environmental effect, the surge, sway and yaw velocity are within the range of  $[0, 6.9]$  m/s,  $[-0.63, 0.44]$  m/s and  $[-2.57, 2.64]$  deg/s, respectively. These velocities in the other dataset are slightly different, ranging within  $[-0.45, 7.3]$  m/s,  $[-1.36, 0.47]$  m/s and  $[-3.58, 3]$  deg/s, respectively. To investigate the influence of environmental factors in ship motion, a wave from east to west, with a significant wave height of 3 meters is applied.

A three-layer ANN with 32 hidden nodes is created for the Garson method; and two five-layer ANNs with hidden nodes 32, 16 and 4, respectively, are constructed for the Morris and Sobol methods. To train the ANN surrogate model, almost 5000 samples are employed. The experiment aims to model ship heading. Therefore, the output of the ANNs is ship heading, and the rest of the parameters, as defined in Table 1, are the inputs.

The experiments are conducted in MATLAB R2018a with a computer equipped

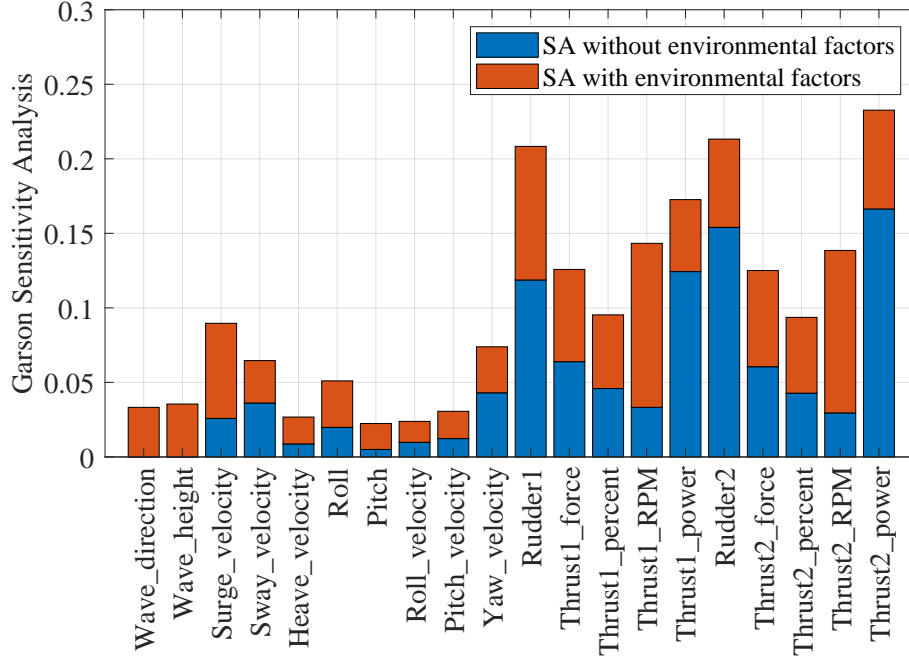


Figure 5: Garson SA results.

with 2.60 GHz i7-6700K CPU and 16 GB RAM. The two widely used global SA methods, Morris and Sobol, are adopt from the SAFE toolbox [37].

#### 4.1. Sensitivity analysis

##### 4.1.1. Sensitivity indices of Garson method

Fig. 5 shows the obtained Garson SA results. In the absence of environmental factors, the most dominant parameters that show a higher Garson value are: the rudder and the power of the two thrusters. While in the case of environmental factors, the most significant factors are RPM of the two thrusters, the rudder of the thruster 1.

There are two other parameters for which the sensitivity index is greater than 0.05 apart from the two most influential parameters, which also indicates significant influence to the model output without environmental factors. However, there are three other parameters for which the sensitivity index is greater



than 0.05 apart from the most important parameters with environmental factors. The parameters whose sensitivity index lower than 0.05 exhibit a very low Garson sensitivity index indicating negligible impact on model output. As shown in Fig. 5, the Garson method detects that most of the parameters have an index of small value ( $\leq 0.05$ ). Due to these smaller values, it is difficult to distinguish a parameter without any effect on the model output.

As Fig. 5 reflects, the sensitivity index of environmental factor is zero in the absence of environmental factors. However, given the environmental factors, the sum sensitivity index of the environmental factors (wave direction and wave height) has reached 0.11. Moreover, under the influence of environmental factors, the position change of the ship has more influence on the heading. In addition, Fig. 5 shows that the Garson method can identify the impact of environmental factors under the influence of environmental factors in the ship motion.

Based on Eq. (2), it should be noted that the Garson method only works for a three-layer neural network. This means that for some complicated data, the three-layer neural network can not achieve sufficient accuracy, which makes Garson's calculation results unreliable.

#### 4.1.2. Sensitivity indices of the Morris method

As previously described, the performance of the Morris method should be examined using a different number of trajectories to determine the number of trajectories needed for a robust ordering of the parameters. Fig. 6 and Fig. 7 summarizes the 10 independent assessments of the Morris method, each of which has 500 trajectories. The  $x$ -axis and  $y$ -axis of Fig. 6 and Fig. 7 represents the mean of elementary effect  $\mu$  and standard deviation  $\delta$ .

In the absence of environmental factors, the highest  $\mu$  values are found for the yaw velocity. Another set of parameters with lower values includes the sway velocity and two rudders. The test parameter wave direction and wave height, which has no effect on the model output, is correctly determined as unimportant with zeros for  $\mu$  and  $\delta$ . In contrast to the results from Garson, the environmental

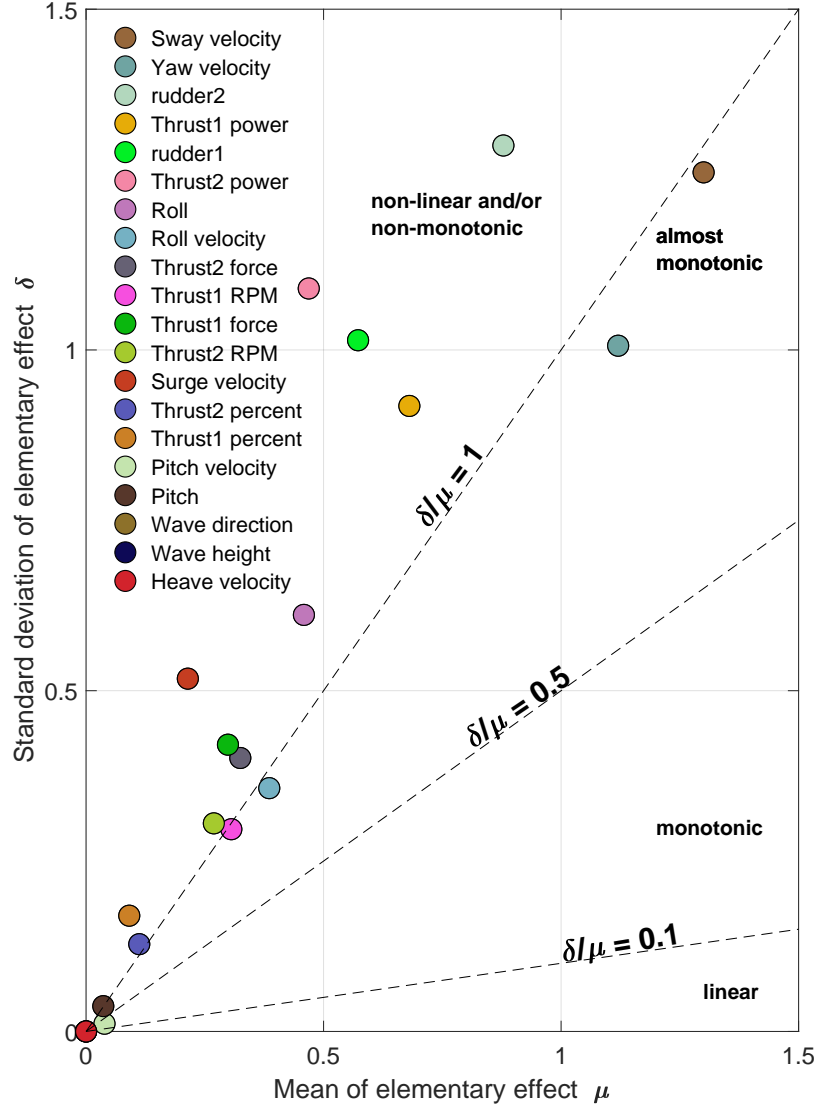


Figure 6: Morris SA for model without environmental factors.

factors have identical influence (0) to the model output.

For the model with environmental factors, the highest  $\mu$  values are also found for the roll, the force of thruster 1, and the power of thruster 2, both of which have lower SA index, follows. The test parameters wave direction and wave height have, no impact on the output. The reason is that in the

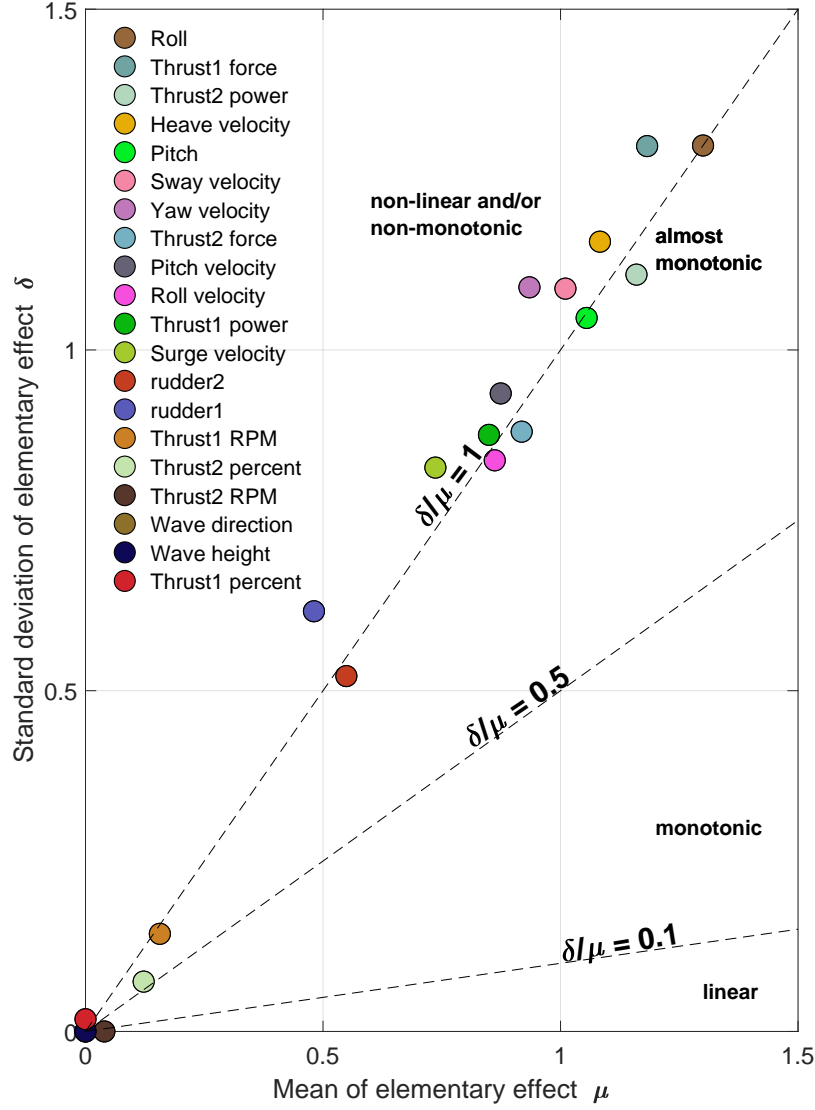


Figure 7: Morris SA for model with environmental factors.

simulated test, the wave direction and wave height are constant. But after the normalization, the constant variable would be changed to 0, which will be considered no influence on the model output by the Morris method.

According to the classification scheme proposed by [38, 39], the ratio  $\delta/\mu$  allows the model parameters to be characterized by (non) linear, (non) mono-

tonic or possibly parametric interactions. For the model without environmental factors, most of the parameters exhibit a ratio  $\delta/\mu \geq 1$ . For example, the rudder 1 and rudder 2 almost have the same ratio in the non-linear area, which indicates these two parameters have a greater non-linear relationship with heading. However, in the model with environmental factors, almost all the parameters exhibit a ratio very close to 1, which indicates that most of the input parameters exhibit almost monotonic behavior, with the interaction of other parameters, or both.

#### 4.1.3. Sensitivity indices of the Sobol method

The first-order sensitivity index  $Si$  and total-order sensitivity index  $STi$  of the Sobol method in the two ship motion with/without environmental factors are illustrated in Fig. 8 and Fig. 9.

As a measure of the importance of input parameters, the first-order sensitivity index  $Si$  identifies that four of the parameters have greater impact on the model output in the ship motion without environmental factors (Fig. 8): sway velocity, yaw velocity, rudder 2, power of the thruster 1 and power of the thruster 2. When the model output is affected by the environmental factors, the first five influential input factors would be: roll, force of thruster 1, power of the thruster 2, pitch, and force of thruster 2, which is shown in Fig. 9.

As a measure of negligible input parameters of model, the total-order sensitivity index identifies four parameters as important in the model without environmental factors (if the threshold is 0.1): sway velocity, yaw velocity, rudder 2, power of the thruster 1, and power of the thruster 2. But there are 15 important input variables in the model with environmental factors which indicate the model is more complicated with the environmental factors. The wave direction and wave height are negligible clearly in both cases from Fig. 8 and Fig. 9. The reason for the influence of wave direction and height under the environmental factors is almost zero is the same with Morris.

Both the first-order sensitivity index  $Si$  and total-order sensitivity index  $STi$  can be used as the indicator for the input selection. Generally, the ranking

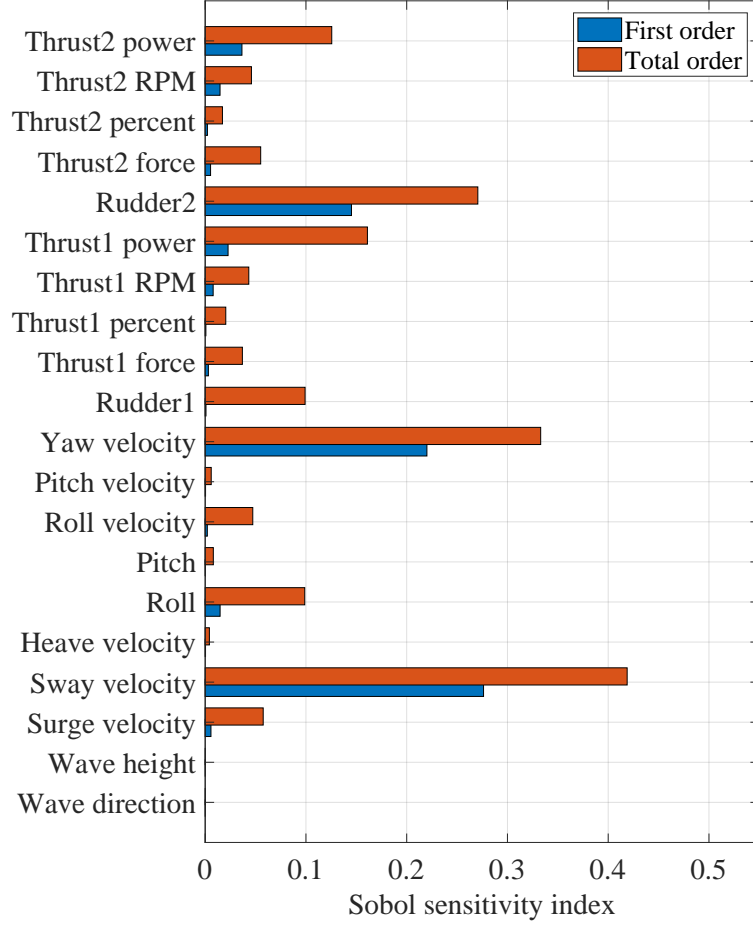


Figure 8: Sobol's SA results without environmental factor.

of total-order sensitivity index  $ST_i$  is similar to the ranking of the first-order sensitivity index  $Si$ . The total-order sensitivity index is selected as the primary indicator to choose input parameters, in contrast to other studies applying the first-order sensitivity index for data-driven modeling, such as Xu et al. [40], who identify 12 parameters of 27 input parameters as influential with  $Si > 0.02$ . The value of the first-order sensitivity index  $Si$  close to zero indicates that these input parameters have no effect on the model output. However, they might have influence on the higher order interactions. Thus, these input parameters might

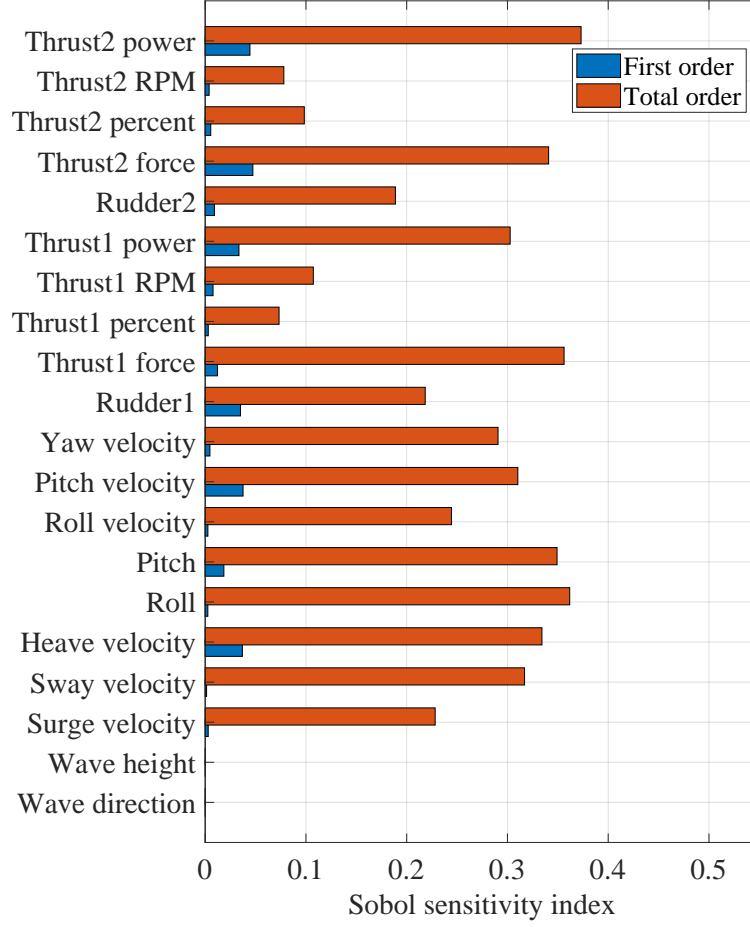


Figure 9: Sobol's SA results with environmental factor.

not be generally negligible. Taking the “thrust1 force” in Fig. 8 and “thrust1 power” in Fig. 9 as an example, the first-order sensitivity index of the two parameters is very small. At the same time, the total-order sensitivity index is relatively larger, which indicates the higher order interactions exist.

#### 4.1.4. Performance comparison of SA methods

Table 2 gives detailed information about the ranking of the three SA methods in the two tests. The numbers in Table 2 represent the ranking order of each variable under different SA methods. For example, the variable “thruster2

Table 2: Ranking of input parameters on the two models for each sensitivity method

| Input             | Testing without environmental factors |          |            | Testing with environmental factors |          |            |
|-------------------|---------------------------------------|----------|------------|------------------------------------|----------|------------|
|                   | Garson                                | Morris   | Sobol(STi) | Garson                             | Morris   | Sobol(STi) |
| wave direction    | 19                                    | 18       | 19         | 13                                 | 18       | 19         |
| wave height       | 19                                    | 18       | 19         | 12                                 | 18       | 19         |
| surge velocity    | 13                                    | 13       | 8          | 6                                  | 12       | 12         |
| sway velocity     | 10                                    | <b>1</b> | <b>1</b>   | 16                                 | 6        | 7          |
| heave velocity    | 17                                    | 18       | 18         | 18                                 | <b>4</b> | 6          |
| roll              | 14                                    | 7        | 7          | 14                                 | <b>1</b> | <b>2</b>   |
| pitch             | 18                                    | 17       | 16         | 19                                 | <b>5</b> | <b>4</b>   |
| roll velocity     | 16                                    | 8        | 10         | 20                                 | 10       | 11         |
| pitch velocity    | 15                                    | 16       | 17         | 17                                 | 9        | 8          |
| yaw velocity      | 8                                     | <b>2</b> | <b>2</b>   | 15                                 | 7        | 10         |
| rudder1           | <b>4</b>                              | <b>5</b> | 6          | <b>3</b>                           | 14       | 13         |
| thruster1 force   | <b>5</b>                              | 11       | 13         | 7                                  | <b>2</b> | <b>3</b>   |
| thruster1 percent | 7                                     | 15       | 14         | 10                                 | 18       | 18         |
| thruster1 RPM     | 11                                    | 10       | 12         | <b>1</b>                           | 15       | 15         |
| thruster1 power   | <b>3</b>                              | <b>4</b> | <b>4</b>   | 11                                 | 11       | 9          |
| rudder2           | <b>2</b>                              | <b>3</b> | <b>3</b>   | 8                                  | 13       | 14         |
| thruster2 force   | 6                                     | 9        | 9          | <b>5</b>                           | 8        | <b>5</b>   |
| thruster2 percent | 9                                     | 14       | 15         | 9                                  | 16       | 16         |
| thruster2 RPM     | 12                                    | 12       | 11         | <b>2</b>                           | 17       | 17         |
| thruster2 power   | <b>1</b>                              | 6        | <b>5</b>   | <b>4</b>                           | <b>3</b> | <b>1</b>   |

power” with a value of 1 indicates the most importance among other variables; whereas the variables “wave direction” with a value of 19 represents a minimum impact on the model output. We use boldface to emphasize the top five most important variables of all methods in Table 2. When compared to the three SA methods in the testing without environmental factors, the ranking of the influential parameters has obtained almost good agreement. It is worth noting that the Garson method still has some inconsistencies with the other two methods, such as those parameters: “thruster2 power”, “rudder 1”. The two environmental parameters: “wave height” and “wave direction”, are identified as non-important parameters, which is consistent with the three SA methods.

Compared with the three SA methods in the ship motion with environmental factors, only the Morris method can rank the parameters in almost the same order as the Sobol method. However, the performance of the Garson method is decreased compared to the other two methods (Morris/Sobol), as this local method is not capable of addressing information in this complex test. An interesting finding of the testing with environmental factors is that the rankings of the environmental parameters (wave direction and wave height) of the three methods are very close.

The parameter ranking of the Morris method and the total-order effects of Sobol is almost the same for most of the parameters in the two data sets, which can be explained by a similar model of parameter variation in the sample matrix for calculation, and both methods are intended to identify non-influential parameters. In addition, a very interesting finding is that the difference between the  $STi$  sensitivity index of each parameter and the difference between the  $\mu$  value of the Morris method is very similar. For almost all parameters, the presence of higher order effects observed in the  $Si$  and  $STi$  indices is also consistent with the high values of the standard deviation  $\delta$  found by the Morris method.

Only the wave factors (wave direction and height) are investigated in the data-driven modeling of ship motion. The two tests (with/without environmental factors) used in the analysis presented in this paper are both based on sensor data sets of offshore operations. However, ship motion is affected by several environmental factors, like winds and currents. Using such a data set may result in incomplete experimental results. In a future study, it would be meaningful to apply a more complex and sophisticated data set of ship motion to investigate whether the Sobol or Morris methods can identify and rank the parameters in the similar order.

#### 4.1.5. Structure optimization of ANN

To verify that our proposed method can be used to optimize the model structure and to understand the influence of different SA methods, we investigate how the input number affect the model output in terms of mean square



Table 3: Comparison of model accuracy of each SA method

| Number of input | Testing without environmental factors |            |            | Testing with environmental factors |         |            |
|-----------------|---------------------------------------|------------|------------|------------------------------------|---------|------------|
|                 | Garson                                | Morris     | Sobol(STi) | Garson                             | Morris  | Sobol(STi) |
| 5               | 0.004                                 | 1.3626e-04 | 0.0936     | 56.4880                            | 9.4673  | 33.9169    |
| 10              | 8.5538e-05                            | 1.4223e-04 | 3.5405e-04 | 6.4942                             | 0.05686 | 0.08489    |
| 15              | 7.1731e-05                            | 7.2146e-05 | 9.2146e-05 | 0.00297                            | 0.00134 | 0.00169    |
| 20              | 7.3383e-04                            | 7.3383e-04 | 7.3383e-04 | 0.00248                            | 0.00248 | 0.00248    |

error. The neural network is re-constructed using the first 5, 10, 15, and 20 most influential input parameters, which are selected by the three SA methods. Each comparison is repeated five times to ensure the predictive convergence. The average comparative result is illustrated in Table 3. Minimum error for the three methods appears when the number of input parameters is 15 in both cases, that is, with and without environmental factors. This indicates that the 15 most influential factors are good enough for ship heading modeling. In addition, the Morris method show superior performance than the other two methods, regardless of the number of input in either cases.

#### 4.2. Uncertainty estimation

As mentioned, the input parameters have its initial conditions and certain ranges in the analysis of traditional mathematical models. In this paper, we characterize the uncertainty of each input parameter by varying all input parameters from their minimum value to their maximum value with uniform distribution. Next, the Latin hyper-cube sampling (LHS) method is employed to generate inputs according to the distributions, and then the well-trained surrogate model would be used to generate model outputs. Throughout numerous iterations, the underlying uncertainties would be represented by the distribution of model outputs based on Eq. (11) and Eq. (12).

The uncertainties of the 20 input parameters in the two ship motion data sets (e.g. with/without environmental factors) are depicted in Fig. 10. In the cases of without environmental factors, the input parameters “sway velocity”, “yaw velocity”, and two rudders are significant uncertainty to the heading. While, in

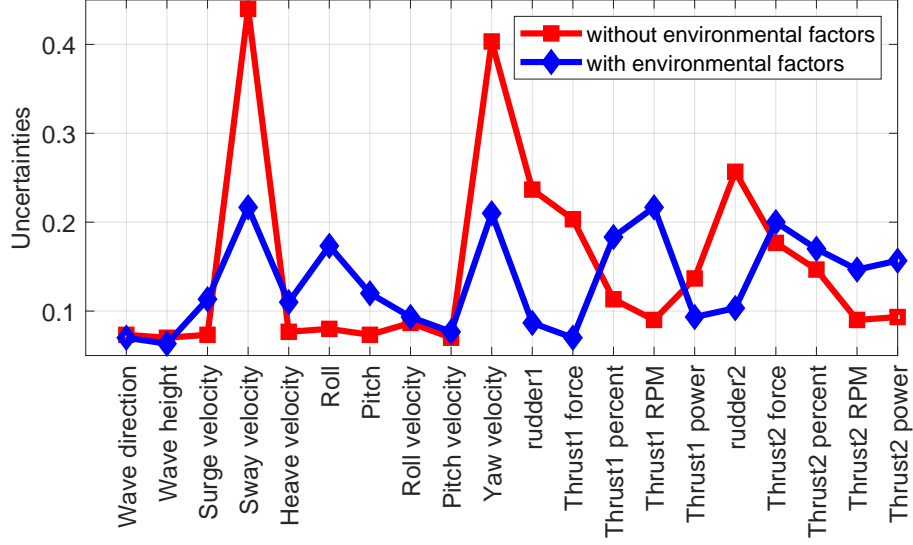


Figure 10: Uncertainties of input parameters in the two datasets.

the case of environmental factors, the five most uncertain parameters are: “sway velocity”, “yaw velocity”, “thruster1 RPM”, “thruster2 force”, and “roll”. When there are no environmental factors, only a few input parameters have uncertain impact on the ship heading. When the environmental factors exist, roll, and force and RPM of the two thrusters have significant uncertainty to the heading. Fig. 11 and Fig. 12 represents the uncertainty of the first six input parameters.

It can be seen from Fig. 10 that the propulsion parameters bring more uncertainty to the heading than those without environmental factors. For example, these parameters “thruster1 percent”, “thruster1 RPM”, “thruster2 force”, “thruster2 percent”, “thruster1 RPM”, and “thruster2 power” are higher. Another noteworthy thing is that the uncertainty of the wave is not obvious, but the uncertainty of the “roll” has changed considerably compared to the situation without the environment. Moreover, the direction of wave is from west to east, and the ship’s heading angle is from south to north. So we can conclude that the uncertainty of the wave is reflected in the “roll”. The reason it is impossible

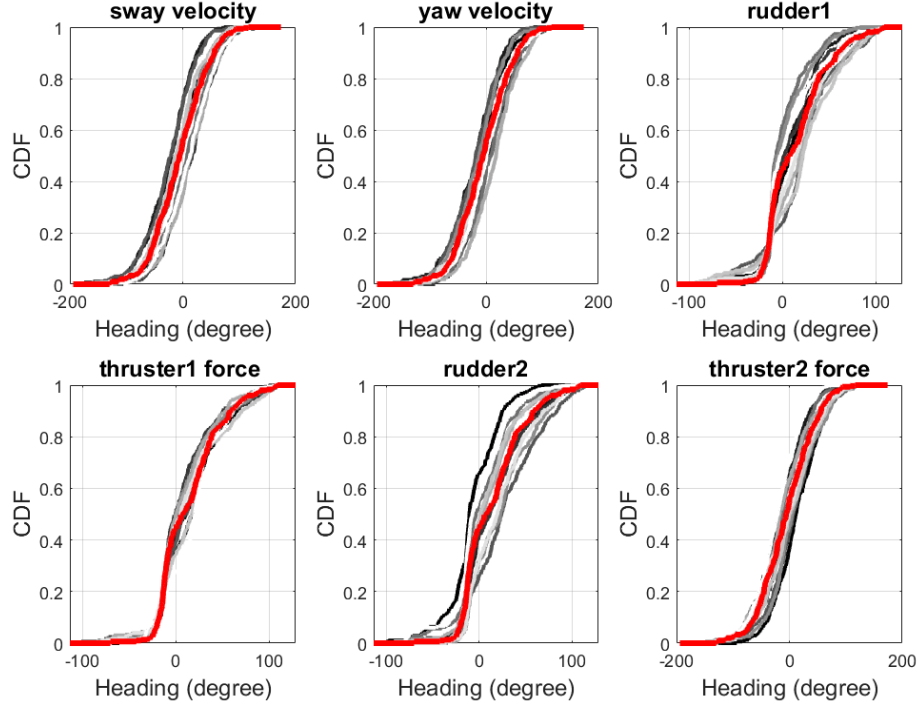


Figure 11: Six uncertain parameters in the dataset without environmental factors.

to get the uncertainty directly from the wave data itself is that the wave is a constant in this experiment, and the constant value is often recognized to be very small by the CDF-based algorithm.

To investigate the uncertainty of the input parameters selected by the three SA methods, we further compare the model output distribution with the first five important parameters removed and the model distribution of all parameters in two datasets. Fig. 13 and Fig. 14 represent the comparison of distribution of model output in the two datasets with and without environmental factors. In this paper, the range of ship heading is from  $-200^\circ$  to  $200^\circ$ . In order to better compare the heading changes in this interval, the PDF plot shows only this range, while the CDF plot shows the whole output range for each methods. From the PDF of Fig. 13, we can see that the five most important parameters

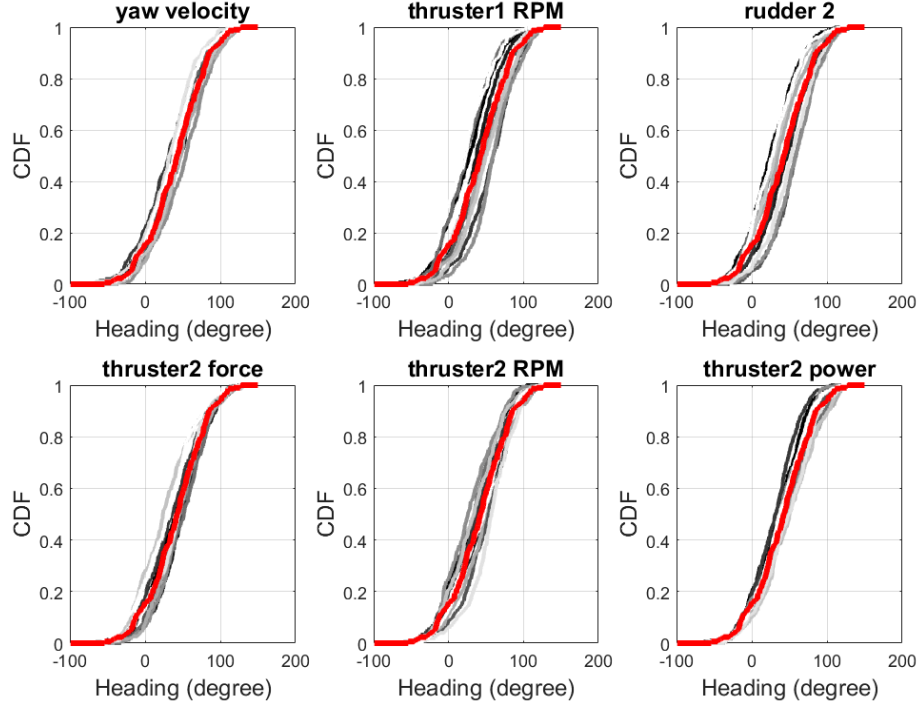


Figure 12: Six uncertain parameters in the dataset with environmental factors.

selected by the Morris method can bring the greatest uncertainty to the model. The uncertainty of the Garson method and the Sobol method in this data set is almost identical. From the CDF in Fig. 13, the same conclusion can be obtained as well. From Fig. 14, the variation of PDF of the three methods is significant. It is difficult to analyze the uncertainty of each method just from the PDF alone. But from the CDF in Fig. 14, it is very clear that Garson's CDF coincides with that of the full parameters model. The greatest uncertainty can be found from the Morris method by calculating the distance between it and the model output of full parameters, even though the CDF of Sobol and Morris vary significantly.

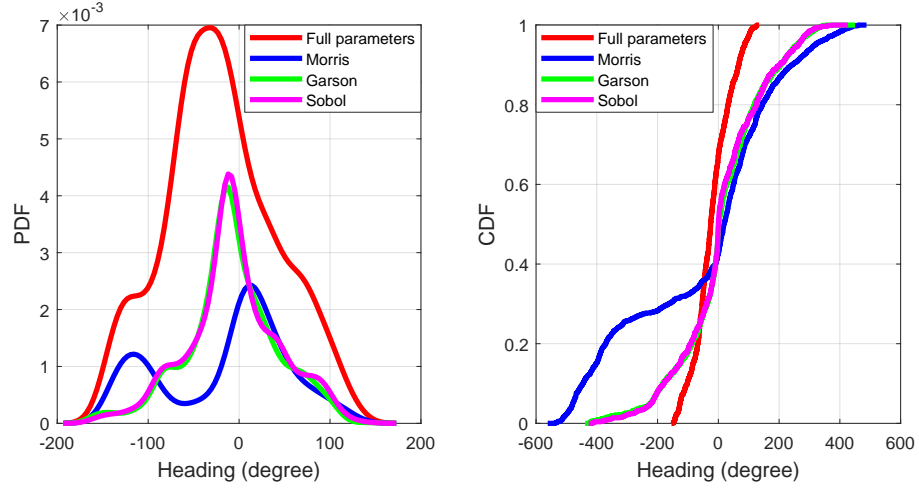


Figure 13: Distribution of model output without environmental factor.

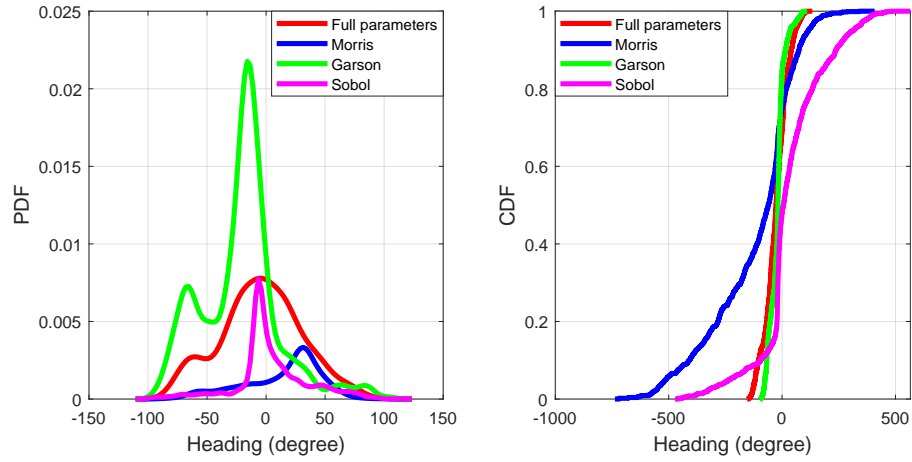


Figure 14: Distribution of model output with environmental factor.

## 5. Conclusion

The paper proposed a framework of sensitivity and uncertainty analysis for ship motion data aiming at the consistent and robust identification of important

parameters. The framework consists of four components: data purification, surrogate modeling, sensitivity and uncertainty analysis methods, and result representation. The proposed framework is flexible for surrogate modeling and sensitivity and uncertainty analysis.

To illustrate the performance of the proposed framework, three SA methods (Garson, Morris, and Sobol) with increasing computational complexity are compared in two ship motion data sets (with and without environmental factors). The three SA methods were conducted on the basis on an ANN model, which is built on the basis of the sensor data of ship motion. The paper demonstrated that the Morris method has almost identical ranking and identification with the Sobol method in both data sets. The Garson method achieves almost the same ranking with the other two methods in the testing of it without environmental factors. But in the testing with environmental factors, the difference of importance ranking becomes larger. The reason for this may be that the system become more complicated in the presence of environmental factors, and the Garson method may not be able to handle this non-linear characteristic. Considering the computational complexity and the ranking of parameters, the Morris method is highly recommended to build a compact data-driven ship motion model. To determine the high degree of uncertainties of a data-driven ship motion model constructed on the basis of sensor data, PDF-based and CDF-based uncertainty analysis methods are employed. The CDF-based method is employed to quantify the uncertainty. The combination of the CDF-based and PDF-based methods is used to evaluate the uncertainty of the group of parameters selected by the SA methods.

The comparison of the three widely used SA methods can provide us a descriptive example of the proposed framework. The comparative results of the three methods provided in this study also can support the assessment and interpretation of the results. This study further shows that the choice of dataset affects the output of all three applied SA methods, mainly in terms of rankings. The above conclusions are highly depending on the data, and the conclusion could be different for another dataset.

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